DETERMINATION OF THERMOPHYSICAL CHARACTERISTICS OF METALS WITH HIGH THERMAL CONDUCTIVITY

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A method is presented for determination of thermophysical characteristics of metals with high thermal conductivity, in which contact thermal resistances are reduced to a minimum.

The determination of thermophysical characteristics of meterials with high thermal conductivity, metals and alloys, under nonstable thermal regimes, is a difficult thermophysical problem. The usual methods used for such determinatons are contact methods. Contact thermal resistance occurs in two different layers; the boundary of the heat detector and the specimen studied, and the boundary of the specimen with the heater surface. The accuracy of the measurements depends on the ratio of the contact resistance to the thermal resistance of the specimen. The lower this ratio, the more accurately will the thermophysical characteristics be determined, other conditions being equal.

For identical specimen size and geometry and identical lubrication of the contacting surfaces, the thermal resistance ratio is significantly higher for good conductors of heat than for poor ones. This then means that the experimental accuracy for highly conductive materials with identical contact conditions is lower than for poor conductors, for example, polymers. To increase the accuracy of such measurements an attempt is usually made to reduce contact thermal resistance by careful processing of the contacting surfaces and use of lubrication. However, theoretical estimates of the effect of contact thermal resistances [1] on the accuracy of thermophysical property determination in highly thermal-conducting materials have shown the insufficiency of the usual means of reducing contact thermal resistance. Thus, for example, for a specimen thickness of $h = 20 \cdot 10^{-3}$ mm and a single layer of Vaseline lubrication $h_0 = 0.01 \cdot 10^{-3}$ m, which corresponds to the highest class of technical surface purity, the error in determination of characteristics of glass, bismuth, and silver by the two temperature–time interval method [2] will be as follows (in %): for glass, $\Delta a/a = 0.4$, $\Delta\lambda/\lambda = 0.3$; for bismuth, 12.6 and 9.0; for silver, 89.5 and 64.0, respectively. Using tap water as the most conductive lubricant, we obtain the following errors (in %): for glass $\Delta a/a = 0.1$, $\Delta\lambda/\lambda = 0.1$, for bismuth, 2.8 and 2.0; for silver, 61.8 and 45.0, respectively.

It follows from analysis of theoretical error calculations that contact thermal resistances are the major factor reducing accuracy in determination of thermophysical characteristics of highly conductive materials.

In developing effective methods for such measurements, an attempt was made to maintain the same experimental technique used in studies of other materials with the two temperature–time interval method [2]. A diagram of the laboratory apparatus is shown in Fig. 1.

With this goal in mind, it was necessary to determine principles governing the temperature field which would permit determination of large values of the parameter ε with sufficient accuracy . In this case it is possible to use elastic or bulk material heat detectors to ensure reliable thermal contact with the solid specimen. To obtain good thermal contact between the solid specimen and the heater, a high power open circulating water heater was used with a low temperature drop between heater and detector. The necessary galvanometer

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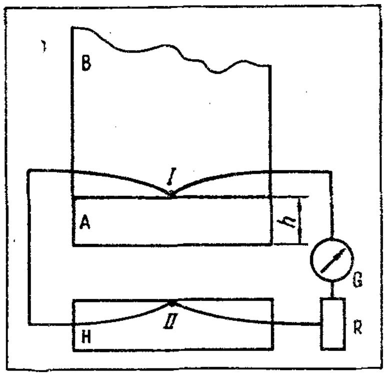

Fig. 1

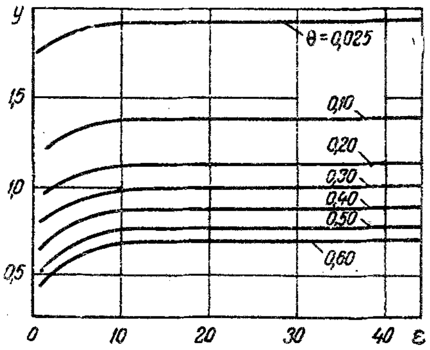

Fig. 2

Fig. 1. Diagram of laboratory apparatus: A) specimen; B) heat detector; H) constant-temperature heater; I and II) differential thermocouple junctions; R) rheostat; G) galvanometer.

Fig. 2. The function $y = f(\varepsilon, \theta)$.

sensitivity may be obtained by increasing the number of junctions in the differential thermobattery.

The two temperature—time interval method is based on the following temperature-field equation:

$$\theta = (1+\alpha)(\operatorname{erfc} y - \alpha \operatorname{erfc} 3y + \alpha^2 \operatorname{erfc} 5y - \ldots), \tag{1}$$

where

$$\alpha = \frac{\varepsilon - 1}{\varepsilon + 1}; \quad \varepsilon = \frac{\lambda}{b\sqrt{a}}; \quad y = \frac{h}{2\sqrt{a\tau}},$$

$$\operatorname{erfc} y = 1 - \operatorname{erf} y; \quad \operatorname{erf} y = \frac{2}{\sqrt{\pi}} \int_0^y e^{-y^2}\, dy. \tag{2}$$

Equation (1) has been tabulated for node points $y = f(\varepsilon, \theta)$ [2]. From analysis of this table it follows that over the interval $0 < \theta < 0.45$ for a change $5 < \varepsilon < \infty$ the value of y is independent of ε with an error from ±1 to ±4%. Thus, the mean y value for $5 < \varepsilon < \infty$ within the limits of the error noted may be taken as a single-valued function of θ, i.e., $y = f(\theta)$. The validity of this statement is supported by Fig. 2.

The thermal diffusivity, a, as follows from Eq. (2), will be determined by the equation

$$a = \frac{h^2}{4\bar{y}^2\tau}, \tag{3}$$

where τ is the time during which the galvanometer light beam indicator moves from initial contact of heater with specimen to a given value $\theta = 1 - N/N_0$ (Fig. 3), and N is the galvanometer indication.

Considering the inconvenience involved in reckoning time τ from the commencement of contact, we may use time intervals $\Delta\tau = \tau_2 - \tau_1$. In this case, we obtain from Eq. (3)

$$a = \frac{h^2}{4\Delta\tau}\left(\frac{1}{\bar{y}_2^2} - \frac{1}{\bar{y}_1^2}\right) = \frac{h^2}{4\Delta\tau} \cdot \frac{1}{\bar{p}}. \tag{4}$$

The values of $1/\bar{p}$ for various θ_1, θ_2 are given in Table 1. To determine the thermal-conductivity coefficient $\lambda = b\,\varepsilon\sqrt{a}$ it is necessary to find the parameter ε. To do this we may use the function $\varepsilon = f(p)$ for given values of θ_1 and θ_2 (Table 2). The value of the argument p is given by

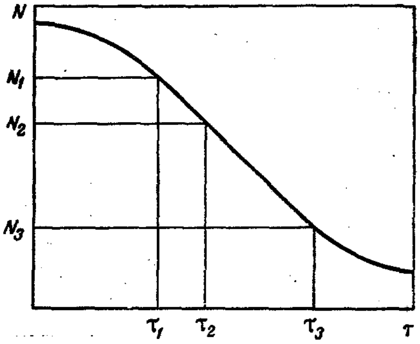

Fig. 3. Function N = f(τ).

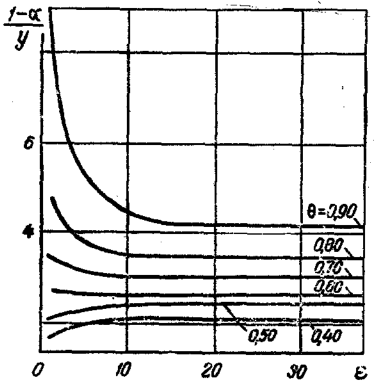

Fig. 4. Function (1 + α)/y = f(ε, θ).

TABLE 1. Function $1/\bar{p} = f(\theta_1; \theta_2)$

θ_2	θ_1				
	0,025	0,05	0,10	0,15	0,20
0,20	0,461	0,378	0,245	0,122	0,000
0,25	0,586	0,503	0,370	0,248	0,126
0,30	0,721	0,638	0,505	0,382	0,260
0,35	0,864	0,781	0,648	0,525	0,403
0,40	1,021	0,938	0,805	0,682	0,560
0,45	1,193	1,110	0,977	0,854	0,738

TABLE 2. Function ε = f(p) for $\theta_1 = 0.30$; $\theta_2 = 0.75$

p	0,17	0,18	0,19	0,20	0,21	0,22	0,23	0,24
ε	2,010	2,103	2,199	2,299	2,402	2,509	2,620	2,737
p	0,25	0,26	0,27	0,28	0,29	0,30	0,31	0,32
ε	2,858	2,986	3,121	3,263	3,414	3,573	3,744	3,925
p	0,33	0,34	0,35	0,36	0,37	0,38	0,39	0,40
ε	4,119	4,328	4,553	4,796	5,060	5,347	5,682	6,008
p	0,41	0,42	0,43	0,44	0,45	0,46	0,47	0,48
ε	6,390	6,815	7,290	7,824	8,430	9,124	9,925	10,850

$$p = \frac{h^2}{4\bar{a}\Delta\tau}. \tag{5}$$

This formula uses the time Δτ, determined from observations at high $\theta_2 \geq 0.60$ since the function ε = f(p) for large θ_i is expressed more clearly than at small θ_i. In Eq. (5) for determination of the argument p the mean value of the diffusivity coefficient $\bar{a}$ is taken, determined in the first part of the experiment from Δτ values for small $\theta < 0.45$. Using the ε = f(p) tables to find ε, the thermal-conductivity coefficient $\bar{\lambda}$ can then be determined:

$$\bar{\lambda} = \bar{b}\bar{\varepsilon}\sqrt{\bar{a}}. \tag{6}$$

We will offer an example of thermophysical property determination for ceramic materials with ε > 5.

The ceramic specimen studied had a thickness h = 14.35 mm. Tire rubber with a thermal activity b = 700 $\mathrm{W \cdot sec^{1/2}/m^2 \cdot deg}$ was used as the thermal detector. The function τ = f(θ) was determined.

TABLE 3. Function $L = f(\theta_1;\ \theta_2)$

θ_2	θ_1								
	0,40	0,45	0,50	0,55	0,60	0,65	0,70	0,75	0,80
0,45	0,742								
0,50	1,086	0,794							
0,55	1,378	1,162	0,848						
0,60	1,640	1,463	1,229	0,889					
0,65	1,908	1,758	1,568	1,319	0,975				
0,70	2,192	2,063	1,904	1,704	1,455	1,080			
0,75	2,498	2,385	2,249	2,083	1,884	1,612	1,198		
0,80	2,882	2,785	2,669	2,531	2,370	2,160	1,871	1,437	
0,85	3,369	3,285	3,189	3,073	2,942	2,776	2,557	2,260	1,744

It was found that at $\theta = 0.10;\ 0.20;\ 0.30;\ 0.40;\ 0.50;\ 0.60;\ 0.70;\ 0.75$ τ, (sec) = 0, 2.8, 5.9, 9.46, 13.8, 19.5, 27.4, 33.0, respectively.

Using Eq. (4) and Table 1, we find $a_{0.10-0.40} = 438 \cdot 10^{-8}$ m^2/sec, $a_{0.10-0.30} = 439 \cdot 10^{-8}$ m^2/sec, $a_{0.20-0.40} = 432 \cdot 10^{-8}$ m^2/sec. The mean value is $a = 436 \cdot 10^{-8}$ m^2/sec.

To determine ε we find with Eq. (5) the values $p_{0.30-0.75} = 0.435$.

For this p value Table 2 gives $\varepsilon = 7.6$. Analogously, from other $\varepsilon = f(p)$ tables, established from the node point table of [2], we find $p_{0.40-0.75} = 0.501$; $\varepsilon = 7.6$; $p_{0.40-0.60} = 1.17$; $\varepsilon = 7.6$; $p_{0.40-0.70} = 0.658$; $\varepsilon = 7.6$; $p_{0.30-0.70} = 0.550$; $\varepsilon = 7.8$. The mean value of $\varepsilon = 7.64$.

Using Eq. (6), we find $\overline{\lambda} = 11.1$ W/m deg.

The value of the parameter ε, appearing in Eq. (6) may be determined in another manner, without recourse to the $\varepsilon = f(p)$ tables. From the node point table $y = f(\varepsilon, \theta)$ it follows that for values of θ, within the interval $0.40 < \theta < 0.85$ and $5 < \varepsilon < \infty$ the ratio $(1 + \alpha)/y$ is independent of ε (Fig. 4), i.e., for each given θ the value $(1 + \alpha)/y = A$ is constant. From this condition it follows that

$$\varepsilon = \frac{hL}{4\sqrt{a\Delta\tau} - hL} = \frac{1}{Q-1}, \tag{7}$$

where

$$L = \sqrt{A_2^2 - A_1^2}\,;\quad Q = \frac{4\sqrt{a}}{h} \cdot \frac{\sqrt{\Delta\tau}}{L}.$$

Numerical values of L for certain values of θ_1 and θ_2 are given in Table 3.

Thus, for example, for $\theta_1 = 0.50$ and $\theta_2 = 0.75$, Table 3 gives a value $L = 2.25$. Consequently, for the observations considered above, we find $\varepsilon = 7.6$.

To verify the validity of the results obtained the ε value may be used to find the value of p, using the table $p = f(\varepsilon)$, and the diffusivity coefficient may be determined from the formula

$$a = \frac{h^2}{4p\Delta\tau_1}.$$

This method permits determination of the diffusivity coefficient a in specimens 20-30 mm thick with an error of 3-5% over the range $2 \cdot 10^{-6} < a < 100 \cdot 10^{-6}$ m^2/sec. Equation (3) or (4), defining the thermal-diffusivity coefficient, is more correct for materials with high diffusivity. This follows from analysis of the function $y = f(\varepsilon;\ \theta)$ (Fig. 2). The thermal-conductivity coefficient λ may be determined with this method to an accuracy of 5-7% in the range $1 < \lambda < 50$ W/m deg for specimens of the same thickness. The thermal conductivity of materials with higher conductivity is determined much more coarsely. In that case the change in $\varepsilon = f(p)$ or $\varepsilon = f(Q)$ becomes quite large in comparison to the change in the argument p or Q. Thus small errors in determining the argument lead to significant errors in ε determination, and consequently, in λ determination. The theory of the method remains valid for any temperature. However, the experimental formulation of the method was designed for determination of thermophysical characteristics in the temperature interval 0-100°C.

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